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A MODEL FOR PREDICTING THE DRY-OUT POSITION FOR ANNULAR FLOW IN A UNIFORMLY HEATED VERTICAL TUBE

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Abstract—The present work introduces a method by which the length of the annular flow regime in a straight vertical-tube steam-generator can be evaluated. The heated length is divided into a large number of segments and the outlet conditions at one segment are used as the initial conditions for the following segment. A computer program has been designed for this step-by-step calculation. A comparison between the results of the present work and different available experimental data demonstrates the adequacy of the presented method.

NOMENCLATURE

C_E, concentration due to entrainment of droplets in the vapour core;§
d, tube diameter;
D, rate of deposition of droplets onto liquid film;
d_D, droplet diameter;
E, rate of entrainment of droplets into vapour core;
f_g, gas friction factor;
f_l, liquid friction factor;
f'_g, interfacial friction factor;
F, the fraction of droplets which are deposited;
F^*, the fraction of droplets which approach the wall;
g, acceleration of gravity, and g_0 gravitational conversion factor;
G, the mass velocity;
h_{fg}, latent heat of vaporisation;
Δh_i, subcooling;
K, the fractional capture of droplets approaching the wall;
k, coefficient of thermal conductivity of the liquid;
$\bar{K}$, mass-transfer coefficient;
l, tube length;
L_B, boiling length;
n, segment number in the numerical solution;
N, number of segments;
P, pressure, and P_f pressure drop due to liquid friction;
$Q_{B.O}$, heat flux at burn-out conditions;
Q/A, heat flux per unit area;
R^+, defined by equation (11);
Re, Reynolds number;
Re_g, gas Reynolds number;
Δr^+, dimensionless annular distance over which the turbulence is sufficiently damped to exert viscous resistance to the penetration of droplets;
S, a dimensionless group defined by equation (8);
T_c, the critical temperature of the fluid substance;
v_g, bulk velocity of vapour;
v_l, bulk velocity of liquid;
$x_{B.O}$, quality at burn-out position;
z, axial distance;
T, liquid temperature;
T_o, a temperature at which σ_o is known.

Greek symbols

α, defined by equation (5);
β, contact angle;
δ, film thickness;
δ_m, mean film thickness;
δ_c, minimum film thickness;
δ_r, relative film thickness, defined in Section 3.5;
δ_s, continuous liquid sublayer thickness;
ρ_l, liquid density;
ρ_g, vapour density;
μ, viscosity;
τ_i, interfacial shear stress;
σ, surface tension;
Γ, defined by equation (15);
σ_o, liquid surface tension corresponding to temperature T_o;
Ψ, ξ, λ, defined by equations (11);
θ, angle between droplet velocity and the tangent to a circle of radius equal to the distance from the axis to the damped layer.

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§This follows the definition adopted in [15].

1. INTRODUCTION

MOTIVATED liquid films flowing in tubes subjected to uniform heating are considered. The present work introduces a method by which the length of the annular flow regime in a straight vertical-tube steam-generator can be evaluated, and hence the dry-out position located.

In earlier works by other investigators, e.g. Barnett [1, 2], Levy [3], Lee and Obertelli [4, 5], Silvestri *et al.* [6], Kirby [7], Hewitt [8] and Hewitt *et al.* [9], attempts were made to correlate the burn-out phenomenon by applying system-describing parameters which were in general of the form:

$$Q_{B.O} = f(d, G, \Delta h_i, l, x_{B.O}).$$

The present work differs from others in that it does not evaluate the overall boiling length by applying a general formula which includes all the factors which affect burn-out. The performance of two-phase flow is very complex during its advance in a steam generator, and we have therefore divided the length of the steam generator tube into segments, and then evaluated the different factors affecting the fluid flow and heat transfer for each segment, the exit conditions of each segment being considered as the inlet conditions to the next. We apply this method to the annular flow regime.

2. THE MODEL

The proposed model is applied to a straight tube (Fig. 1) which is uniformly heated and in which the fluid flows. Only the annular two-phase flow regime is considered; the two-phase friction pressure drop, the liquid and the gas phase friction factor, interfacial shear force and surface wave effects are all considered, while the effect of nucleate boiling is neglected. The idea is that, given the inlet water and steam flow rates, and the operating conditions, the dry-out position can be predicted as a function of the applied heat flux, rate of droplet entrainment and rate of droplet deposition. To locate the dry-out position, a step-by-step calculation along the heated tube is carried out starting from the inlet section and with the end boundary condition either that the film thickness is less than or equal to the critical film thickness, or that the film flow rate equals zero. At each step (location) in the heated tube, the following parameters are determined: liquid flow rate in the film, steam flow rate, liquid and vapour properties, liquid and vapour Reynolds numbers, film thickness, pressure gradient, liquid and vapour friction factor, interfacial friction factor, free film surface roughness (i.e. wave amplitude), and the interfacial shear stress.

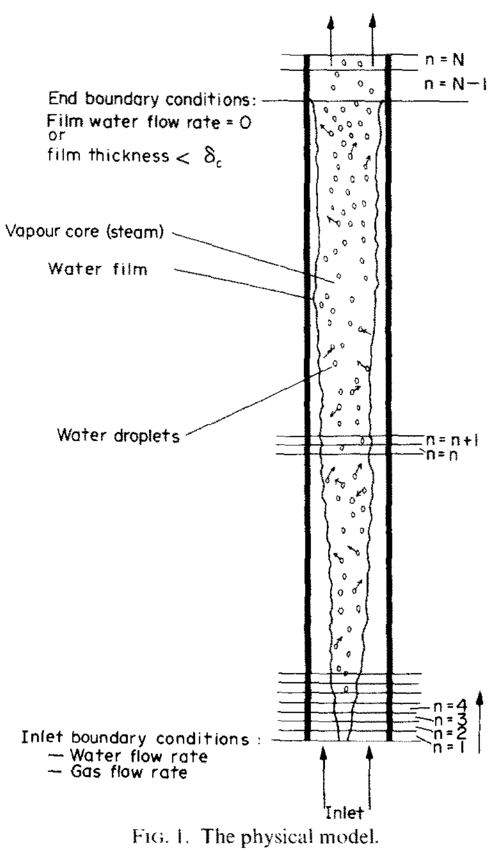


FIG. 1. The physical model.

3. THE RELATIONS USED IN CALCULATING THE DIFFERENT PARAMETERS

3.1

The Lockhart–Martinelli [10] correlation is used for calculating the two-phase friction pressure drop:

$$\phi_{g \text{ or } l} = \left(\frac{(\mathrm{d}P_f/\mathrm{d}z)_{TP}}{(\mathrm{d}P_f/\mathrm{d}z)_{g \text{ or } l}}\right)^{1/2} \tag{1}$$

$$X = \left(\frac{(\mathrm{d}P_f/\mathrm{d}z)_l}{(\mathrm{d}P_f/\mathrm{d}z)_g}\right)^{1/2} \tag{2}$$

where $(\mathrm{d}P_f/\mathrm{d}z)_l$ and $(\mathrm{d}P_f/\mathrm{d}z)_g$ are the pressure gradients for the vapour and liquid phases, respectively, flowing alone in single-phase flow within the channel.

The relation between X and ϕ is given graphically; by calculating X, the value of ϕ can be evaluated; hence, the two-phase friction pressure gradient $(\mathrm{d}P_f/\mathrm{d}z)_{TP}$.

3.2

$(\mathrm{d}P_f/\mathrm{d}z)_l$ and $(\mathrm{d}P_f/\mathrm{d}z)_g$ are calculated from the relation:

$$(\mathrm{d}P_f/\mathrm{d}z)_l = 4\frac{f_l}{d}\left(\tfrac{1}{2}\rho_l v_l^2\right) \tag{3}$$

$$(\mathrm{d}P_f/\mathrm{d}z)_g = 4\frac{f_g}{d}\left(\tfrac{1}{2}\rho_g v_g^2\right) \tag{4}$$

where f_l and f_g are the liquid and vapour friction factors and v_l and v_g are the liquid and vapour velocity, respectively.

The friction factors f_l and f_g are given: for Re up to 2000, $f = 16Re^{-1}$; and for Re greater than 2000, $f = 0.79Re^{-0.25}$.

3.3

Because of the inter-relationship between liquid-film flow, liquid-film thickness and pressure gradient, if any two of these factors are known, the third can be calculated.

The film thickness is evaluated applying the relationship proposed by Turner and Wallis [11]:

$$(1-\alpha) = \left[\frac{(dP_f/dz)_l}{(dP_f/dz)_{TP}}\right]^{1/2} \quad (5)$$

where $(1-\alpha) \simeq 4\delta/d$, for a small film thickness δ compared to tube diameter d.

3.4

The force balance on a tube section of length Δz is calculated by considering the friction pressure gradient $(\Delta P/\Delta z)_{TP}$ in relation to the interfacial shear stress for fully-developed flow by the equation:

$$\tau_i = \left[\frac{g}{g_o}\rho_g - \left(\frac{\Delta P}{\Delta z}\right)_{TP}\right]\left(\frac{d}{4} - \frac{\delta_m}{2}\right). \quad (6)$$

3.5

In order to obtain the interfacial roughness $(\delta_m - \delta_s)$,* we used Gill *et al.* [12, 13] correlation between interfacial roughness and relative film thickness defined by $\delta_r = \delta - (5d/Re_g)(2/f_g)^{1/2}$.

3.6

The Chien *et al.* [14] relation between the mean height of the disturbed layer and the superficial friction factor is used:

$$f_g' = 1.2\left(\frac{\delta_m - \delta_s}{d}\right)^{0.485} \quad (7)$$

where d is the diameter of the tube, and δ_m is the mean film thickness.

3.7

The rate of droplet entrainment to the vapour core is calculated applying Hutchinson and Whalley's [15] hypothesis for entrainment; the rate of entrainment is represented by the dimensionless group S:

$$S = \frac{\tau_i \delta}{\sigma} \quad (8)$$

where τ_i is interfacial shear stress, δ is film thickness, and σ is liquid surface tension. Having calculated S, the corresponding droplet entrainment concentration was evaluated from a curve fit applied to Hutchinson *et al.* [15] results. We then evaluated entrainment rate (E) from the equation:

$$E = C_E\bar{K} \quad (9)$$

where $\bar{K}$ is the mass-transfer coefficient (estimated to be 0.014 m/s for data similar to Hewitt *et al.* [9] experiments).

3.8

The analyses of Hutchinson *et al.* [16] for droplet deposition rate are followed. The deposition of the droplets is based on the assumptions that the droplets do not move with the eddies, their motion is stochastic and only droplets with sufficient momentum are deposited. Thus:

$$F = KF^* \quad (10)$$

where F is the fraction of droplets which are deposited, F^* is the fraction of droplets which approach the wall, and K is the fractional capture of droplets approaching the wall:

$$\left.\begin{aligned} K &= \int_{-\pi/2}^{\pi/2} e^{-\Psi}\,d\theta \\ \Psi &= 648\frac{(\rho_g/\rho_l)^2(\Delta r^+)^2}{f_g^2[Re_g(d_D/d)]^4}\xi^2 \\ \xi &= \frac{(1-\lambda^2\sin^2\theta)^{1/2} - \lambda\cos\theta}{(1-\lambda)} \\ \lambda &= 1 - \frac{\Delta r^+}{R^+} \\ R^+ &= Re_g\frac{f_g}{8} \end{aligned}\right\} \quad (11)$$

where θ is the angle between droplet velocity and the tangent to a circle of radius equal to the distance from the axis to the damped layer, f_g is the vapour friction factor, Re_g is the vapour Reynolds number, d_D is droplet diameter, d is tube diameter, ρ_l is droplet liquid density, ρ_g is vapour density, and Δr^+ is dimensionless annular distance over which the turbulence is sufficiently damped to exert viscous resistance to the penetration of droplets ($\Delta r^+ = 1.25$ showed an excellent fit for all experimental data discussed by Hutchinson *et al.* [16]). F^* was evaluated from the knowledge of the vapour phase Reynolds number, density ratio of the vapour phase and the liquid droplets, and droplet diameter (Hutchinson *et al.* [16]). The calculation procedure for F^* is given in detail in [17]. An average droplet diameter of 30μ was considered. Hence, F is calculated from equation (10) and the rate of droplet deposition (D) is then evaluated from the equation:

The rate of droplet deposition = fraction of droplets which are deposited $(F) \times$ rate of entrainment of droplets into the vapour core. Hence:

$$D = FC_E\bar{K}. \quad (12)$$

It should be noted that at the first section of the annular region, the droplet concentration in the vapour is that due to local entrainment at this section. In the subsequent sections, the droplet concentration in the vapour core is the cumulative concentration from the previous sections plus droplets entrained locally less those deposited (see Fig. 1).

3.9

The local film flow rate was calculated at the end of each segment of the heated tube by applying the following mass balance equation:

$$d\,G_{\text{film}} - \frac{4}{d}\left[D - E - \left(\frac{Q}{A}\right)\Big/ h_{fg}\right]dZ = 0. \quad (13)$$

*This is usually defined as $(\delta_m - \delta_s)$ where δ_m and δ_s are, respectively, the mean film thickness and the continuous sublayer thickness. The Gill *et al.* correlation gives $(\delta_m - \delta_s)$ directly and it is not necessary to evaluate δ_m and δ_s separately. A single symbol could have been used, but we prefer to retain the existing nomenclature.

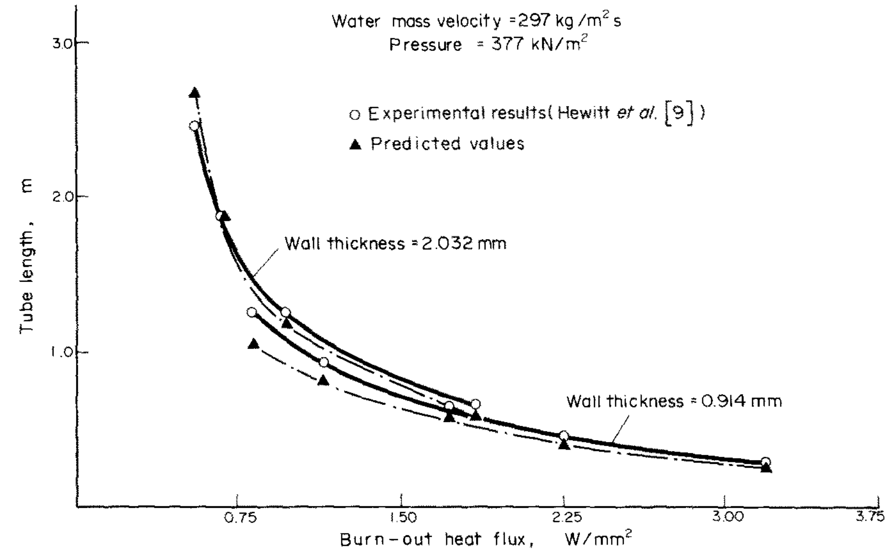


FIG. 2. Effect of heat flux on the position of burn-out point in a uniformly heated tube.

3.10

The critical film thickness below which the liquid film would break up was calculated from the analysis presented by Zuber and Staub [18]:

$$\frac{\rho_l}{6}\left(\frac{2\Gamma}{\rho_l\delta_c}\right)^2 = \frac{\sigma(1-\cos\beta)}{\delta_c} + \frac{\partial\sigma}{\partial T}\frac{(Q/A)}{k}\cos\beta + \rho_g\left(\frac{Q/A}{\rho_g(h_g-h_l)}\right)^2\frac{(\rho_l-\rho_g)\cos^2\beta}{\rho_l} \quad (14)$$

where:

$$\Gamma = \frac{\rho_g\tau_i}{2\mu}\delta_c^2. \quad (15)$$

The surface tension gradient ($\partial\sigma/\partial T$) could be evaluated from the formula:

$$\sigma = \sigma_o\left(\frac{T_c-T}{T_c-T_o}\right)^{1.2}$$

where T_c is the critical temperature of the fluid substance, and σ_o is the surface tension corresponding to temperature T_o. Hence:

$$\frac{d\sigma}{dT} = -1.2\sigma_o\frac{(T_c-T)^{0.2}}{(T_c-T_o)^{1.2}}. \quad (16)$$

Thompson [19] has found that the contact angle for steam–water flow is independent of steam and water rates in the limited range of flow conditions under which a stable interface can exist, hence the contact angle (β) in the present work is taken to be 64.8°.

4. THE PROGRAMME STRATEGY

The programme is designed on the principle of a step-by-step calculation. The heated length is divided into a large number of segments (N). At each segment the different parameters are calculated. The outlet conditions at segment n (vapour flow rate, liquid flow rate in the film and concentration of liquid droplets in the vapour core) are used as initial conditions for the following segment ($n+1$).

5. BOUNDARY CONDITIONS

The inlet boundary conditions are the water and steam flow rates. We have chosen inlet boundary conditions similar to available experimental results (e.g. $x = 0.4, 0.42, 0.44$). The end boundary condition is that the liquid film thickness has reached a value equal to or less than the critical film thickness. Equation (13) is solved to give the value of the critical film thickness (δ_c) which is used as the end boundary condition in the present work.

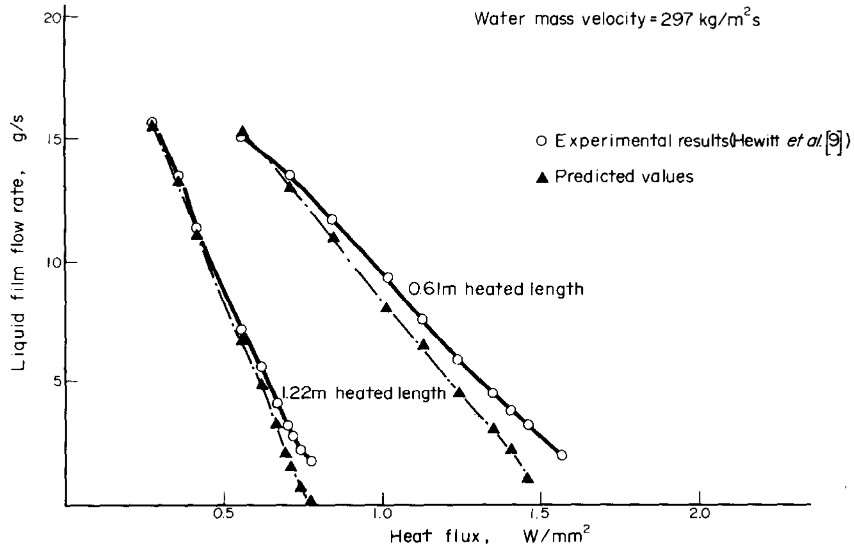
Subroutines and functions are attached to the main program (El-Shanawany [17]) to evaluate the rate of droplet entrainment, to evaluate the Lockhart–Martinelli parameter ϕ, to calculate wave amplitude on the continuous liquid sub-layer thickness, to determine the thermodynamic properties of the vapour and to plot the results.

6. RESULTS AND DISCUSSION

The results of the present work are compared with the experimental data reported in [9].

Hewitt *et al.* [9] used a test section which consisted of an electrically heated tube 3.05 m long and 9.35 mm dia with wall thickness of either 0.92 mm (thin tube) or 2.03 mm (thick tube). Steam and water flowed inside the tube at pressures ranging from 0.1 to 0.4 MN/m^2. The same experimental conditions were applied to our model. The relation between the burn-out heat flux and the boiling tube length for thin and thick tubes as calculated by the present analysis and as found by Hewitt *et al.* [9] is shown in Fig. 2. It can be seen that there is fair agreement between the experimental results and the values evaluated by the present analysis.

Figure 3 shows the relation between the liquid film flow rate at the exit of the heated tube when burn-out conditions were not reached, and the applied heat flux. Again, there is close agreement between the experiments of [9] and the present predicted values.



3. The relation between the liquid film flow rate at the exit of the heated tube and the applied heat flux.

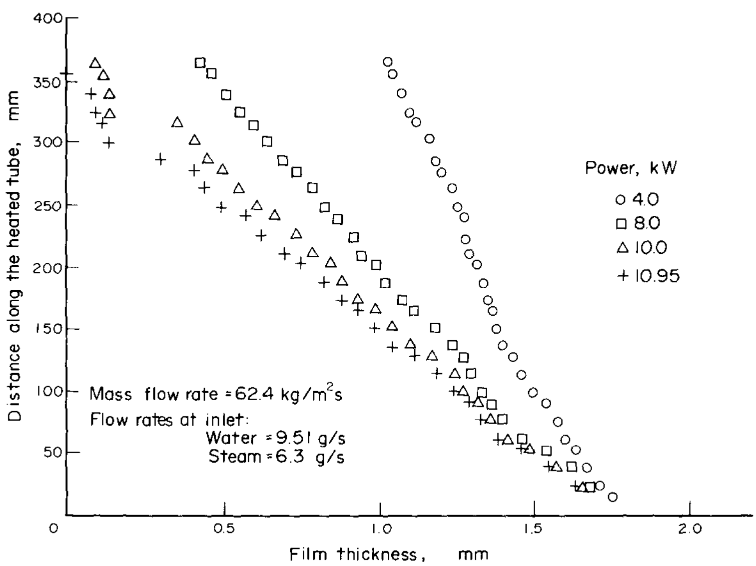


FIG. 4. Change in film thickness with distance along the heated tube (predicted).

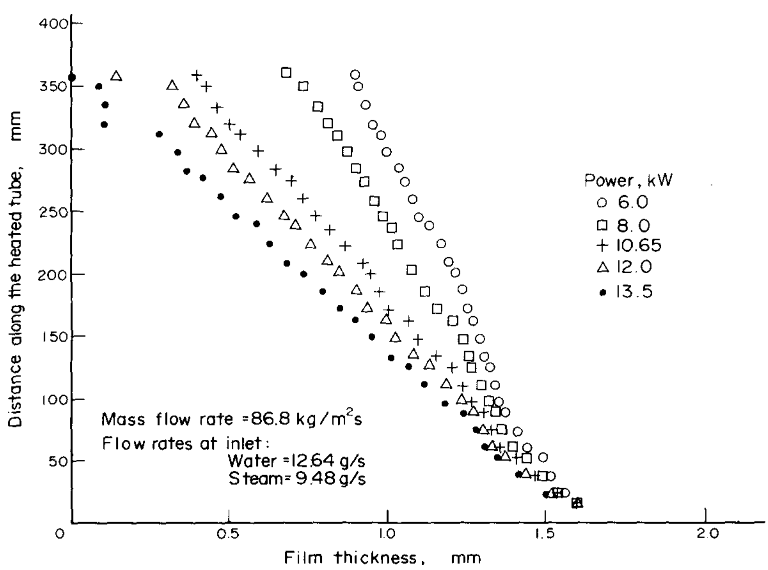


FIG. 5. Change in film thickness with distance along the heated tube (predicted).

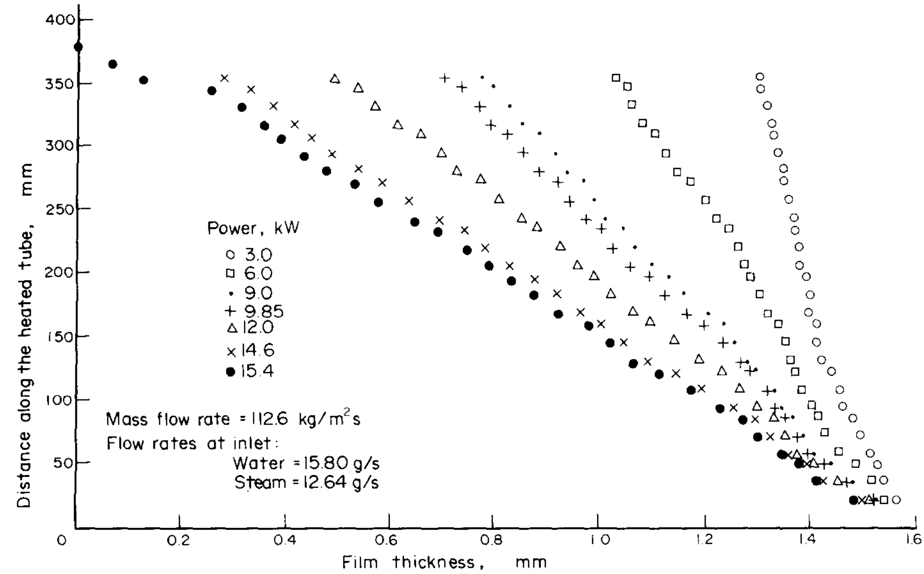


FIG. 6. Change in film thickness with distance along the heated tube (predicted).

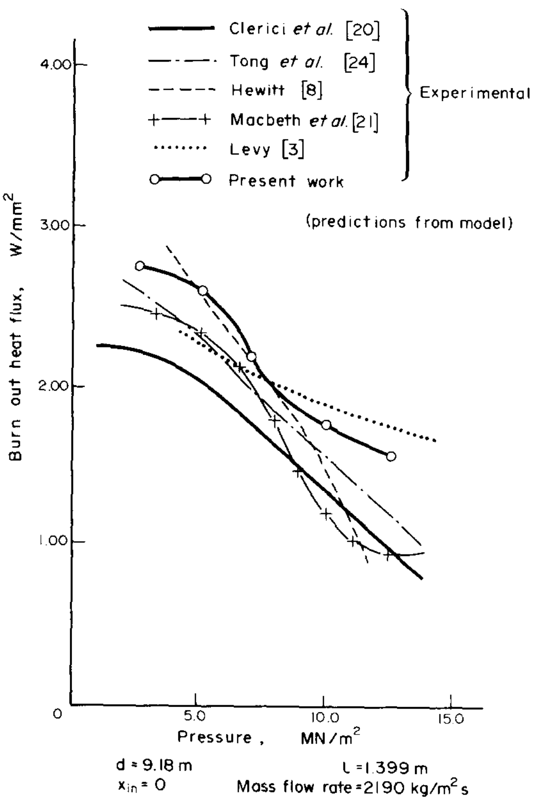


FIG. 7. The relation between burn-out heat flux and the operating pressure.

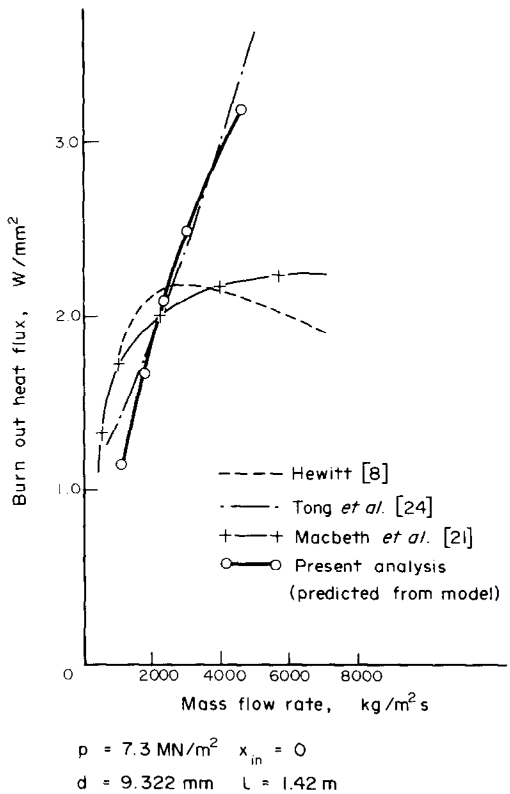


FIG. 8. Relation between burn-out heat flux and mass flow rate.

Applying the computer program "BURN" [17], we calculate the change in the film thickness with distance along the heated tube for three total mass flow rates, 62.4, 86.8 and 112.6 kg/m^2 s, with tube heating power being a parameter (Figs. 4–6). As the heating power increases, the film thickness decreases at the same position on the tube. When higher heating powers are applied, the film thickness continues to decrease gradually with distance, and just before the dry-out a sudden decrease in the film thickness occurs.

As a further step towards the examination of the present analysis, our results are compared with burn-out correlations collected by Clerici *et al.* [20], Levy [3], Hewitt [8], Macbeth and Thompson [21], Adorni *et al.* [22], Gaspari *et al.* [23] and Tong *et al.* [24]. With reference to Fig. 7, which shows the relation between the burn-out heat flux and the operating pressure, it can be seen that the results of the present investigation give higher values of burn-out flux than those of Macbeth *et al.* [21], Tong *et al.* [24],

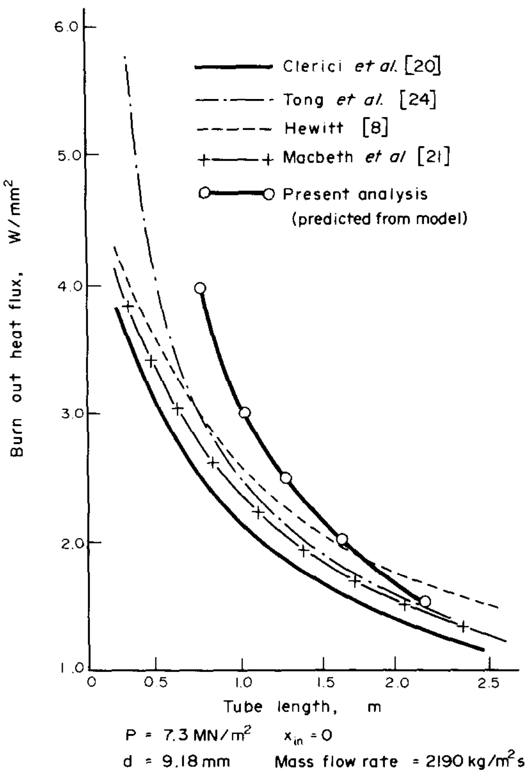


FIG. 9. Graphical comparison with burn-out correlations.

and Gaspari *et al.* [23], for the same conditions, $d = 9.18$ mm, $l = 1.393$ m, $x_{in} = 0$ and mass velocity $= 2190\,kg/m^2\,s$.

Figure 8, which shows the relation between heat flux and mass flow-rate, illustrates that for the same operating conditions, the present work predicts burn-out values close to those of Tong *et al.* [24]. Nevertheless, the results of Macbeth *et al.* [21] and Hewitt [8] showed different trends. Their results show that the burn-out heat flux reaches a maximum value after which it tended to decrease with mass flow-rate.

In Fig. 9, the relation between burn-out heat flux and tube length is shown for the present work and for other investigations, where the operating conditions are the same for all the curves ($P = 7.3\,MN/m^2$, $d = 9.18$ mm, $x_{in} = 0$ and mass velocity $= 2190\,kg/m^2\,s$). For longer tubes, the present results show a fair agreement with other workers' results, while for shorter tubes, the present analysis results in a higher burn-out heat flux than that found by other workers, namely Hewitt [8], Macbeth *et al.* [21], Tong *et al.* [24], Adorni *et al.* [22], and Gaspari *et al.* [23]. It could be that for shorter tubes, the entrainment process has not reached a developed condition.

REFERENCES

1. P. G. Barnett, An investigation into the validity of certain hypotheses implied by various burn-out conditions, AEWW-R 214 (1963).
2. P. G. Barnett, A correlation of burn-out data for uniformly heated annuli and its use for predicting burn-out in uniformly heated rod bundles, AEWW-R (1966).
3. S. Levy, Prediction of the critical heat flux in forced convection flow, GEAP 3961 (1962).
4. D. H. Lee and J. D. Obertelli, An experimental investigation of forced convection burn-out in high pressure water. Part I: Round tubes with uniform flux distribution, AEEW-R 213 (1963).
5. D. H. Lee and J. D. Obertelli, An experimental investigation of burn-out with forced convection high-pressure water, *Proc. Instn Mech. Engrs* **180**, 27 (1966).
6. M. Silvestri, A research programme in two-phase flow, CANI-CISE (1963).
7. G. J. Kirby, A model for correlating burn-out in round tubes, AEEW-R 511 (1966).
8. G. F. Hewitt, A method of representing burn-out data in two-phase heat transfer for uniformly heated round tubes, AERE-R 4613 (1964).
9. G. F. Hewitt, P. M. C. Lacey and D. J. Pulling, Burn-out and film flow in the evaporation of water in tubes, *Proc. Instn Mech. Engrs* **180**, Part 3C (1965).
10. R. W. Lockhart and R. C. Martinelli, Proposed correlation of data for isothermal two-phase, two-component flow in a pipe, *Chem. Engng Prog.* **45**, 39 (1949).
11. J. M. Turner and G. B. Wallis, An analysis of the liquid film in annular flow, NYO-3114-13 (1965).
12. L. E. Gill, G. F. Hewitt, P. Hutchinson and P. M. C. Lacey, Sampling probe studies of the gas core in annular two-phase flow. I: The effect of length on phase and velocity distribution, *Chem. Engng Sci.* **18**, 525 (1963).
13. L. E. Gill, G. F. Hewitt and P. M. C. Lacey, Sampling probe studies of the gas core in annular two-phase flow, II: Studies of the effect of phase flow rate on phase and velocity distribution, AERE-R 3955 (1963).
14. S. F. Chien and W. Ibelle, Pressure drop and liquid film thickness of two-phase annular and annular mist flows, *J. Heat Transfer* **86C**(1), 89–96 (1964).
15. P. Hutchinson and P. B. Whalley, A possible characterisation of entrainment in annular flow, AERE-R 7126 (1972).
16. P. Hutchinson, G. F. Hewitt and A. E. Dukler, Deposition of liquid or solid dispersions from turbulent gas stream: a stochastic model, *Chem. Engng Sci.* **26**, 419 (1971).
17. M. El-Shanawany, The flow of heated liquid films with special reference to dry-out, Ph.D. Thesis, University of London (1974).
18. N. Zuber and F. W. Staub, Stability of dry patches forming in liquid films flowing over heated surfaces, *Int. J. Heat Mass Transfer* **9**, 897 (1966).
19. T. S. Thompson, Liquid film breakdown and dry-patch stability of steam-water flows with heat transfer, Ph.D. Thesis, University of London (1970).
20. E. C. Clerici, S. Garriba, R. Sala and A. Tozzi, A catalogue of burn-out correlation for forced convection in the quality region, EUR 3300e (1966).
21. R. V. Macbeth and B. Thompson, Burn-out in uniformly heated round tubes: A compilation of world data with accurate correlations, AGEW-R 356 (1964).
22. N. Adorni *et al.*, Phase and velocity distribution measurements in a round vertical tube, CISE R-91 (1964).
23. G. P. Gaspari *et al.*, Pressure drops in steam–water mixtures, CISE R-83 (1964).
24. L. S. Tong *et al.*, *DNB (Burn-Out) Studies in an Open Core*, Westinghouse Electric Corporation, WCAP-3736, Volumes I and II (1964).

UN MODELE POUR PREDIRE LA POSITION DE L'ASSECHEMENT POUR UN ECOULEMENT ANNULAIRE DANS UN TUBE VERTICAL ET UNIFORMEMENT CHAUFFE

Résumé—L'article présente une méthode dans laquelle on peut évaluer la longueur du régime d'écoulement annulaire dans un tube droit et vertical de générateur de vapeur. La longueur chauffée est divisée en un grand nombre de segments et les conditions de sortie d'un segment sont les conditions d'entrée pour le segment suivant. Un programme informatique a été conçu pour ce calcul pas-à-pas. Une comparaison entre les résultats de ce travail et différents résultats expérimentaux montrent la validité de la méthóde présentée.

EIN MODELL ZUR VORHERSAGE DES DRY-OUT-PUNKTES BEI RINGSTRÖMUNG IN EINEM GLEICHMÄSSIG BEHEIZTEN SENKRECHTEN ROHR

Zusammenfassung—Die Arbeit stellt eine Methode vor, mit welcher die Länge des Ringströmungsbereichs in einem Dampferzeuger mit geraden, senkrechten Rohren bestimmt werden kann. Die beheizte Länge wird in eine große Anzahl Abschnitte unterteilt und die Austrittsbedingungen eines Abschnitts werden als Anfangsbedingungen des folgenden Abschnitts benutzt. Es wurde ein Computerprogramm für diese schrittweise Berechnung aufgestellt. Ein Vergleich der hiermit erzielten Ergebnisse mit verschiedenen verfügbaren experimentellen Daten zeigt die Angmessenheit der vorgestellten Methode.

МОДЕЛЬ РАСЧЕТА ЗОНЫ ВЫСУШИВАНИЯ ПРИ ТЕЧЕНИИ В КОЛЬЦЕВОМ КАНАЛЕ РАВНОМЕРНО НАГРЕВАЕМОЙ ВЕРТИКАЛЬНОЙ ТРУБЫ

Аннотация — Предложен метод расчета длины кольцевого зазора в прямоточном парогенераторе, выполненном в виде вертикальной трубы. Нагреваемый участок разделен на большое число участков, а условия на выходе на предыдущем участке используются как начальные условия на следующем. Для проведения последовательного расчета была составлена вычислительная программа. Сравнение результатов данного исследования с различными имеющимися экспериментальными данными свидетельствует об адекватности представленного метода.